[Another look at] decay in astrophysical neutrinos
(work in progress)

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Introduction

- The very long baselines of astrophysical ν's ($\gtrsim$ 50 Mpc) make them ideal for studying the possibility that they are unstable and decay
- We will focus on the redshift dependence of the decay
- For a population of mass eigenstates,

$$\frac{dN}{dt} = -\lambda N \qquad \text{with} \qquad \lambda = \frac{1}{\tau} = \frac{m}{\tau_0 E} \equiv \frac{\kappa}{E}$$

where

τ_0 : lifetime in the rest frame

τ : lifetime in the lab frame

t : time ellapsed since creation (in lab frame)

$\kappa \equiv m/\tau_0$

- Lower bounds on $\kappa^{-1} \equiv \tau_0/m$:
  - For ν_1: $\kappa_1^{-1} \gtrsim 10^5$ s eV^{-1} (from SN1987A)
  - For ν_2: $\kappa_2^{-1} \gtrsim 10^{-4}$ s eV^{-1} (from solar ν's)
  - For ν_3: $\kappa_3^{-1} \gtrsim 10^{-10}$ s eV^{-1} (from atm. and long-baseline ν's)

Two ways of solving the decay equation

$$\frac{dN}{dt} = -\lambda N$$

for astrophysical ν's:

1. *Naive* solution: introducing the redshift dependence after solving the equation (usual way)
2. *Proper* (we think!) solution: introducing the redshift dependence before solving the equation

These lead to very different behaviours for the surviving population $N(z)$ of neutrinos created at redshift z.

Also, it is commonly assumed that the decay of cosmological ν's is complete when they reach Earth – we have found some caveats.

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The *naive* solution

Step 1: Forget about the z-dependence and solve for N as usual:

$$\frac{dN}{dt} = -\lambda N \Rightarrow N(t) = N_0 e^{-\lambda t} \xrightarrow{L=ct} N(L) = N_0 e^{-\lambda L}$$

Step 2: Now, introduce the z-dependence through:

- $L =$ light-travel distance $= L(z)$

$$e^{-\lambda L} \longrightarrow e^{-\lambda L(z)}$$

- relation between energy at production (E) and at detection (E_0): $E = (1+z)E_0$

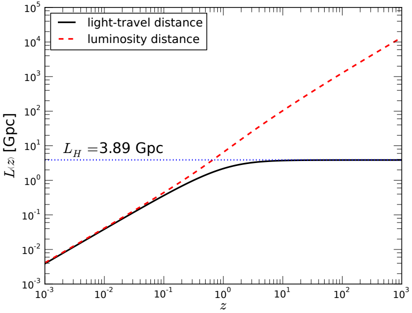
$$\lambda = \frac{\kappa}{E} \longrightarrow \lambda(z) = \frac{\kappa}{E_0(1+z)}$$

So, the naive solution is

$$N_{z,1}(E_0, z) = N_0 \exp\left(-\frac{\kappa}{E_0}\frac{L(z)}{(1+z)}\right)$$

Two observations

1. The light-travel distance is bounded by the Hubble horizon:



So, $e^{-L(z)}$ does not reach zero, even when $z \to \infty$.

2. For a fixed E_0, the decay constant $\lambda(z) = \dfrac{\kappa}{E_0(1+z)}$ will be lower for more distant sources. This helps ν's from farther away to have a higher chance of surviving.

Behaviour of the naive solution

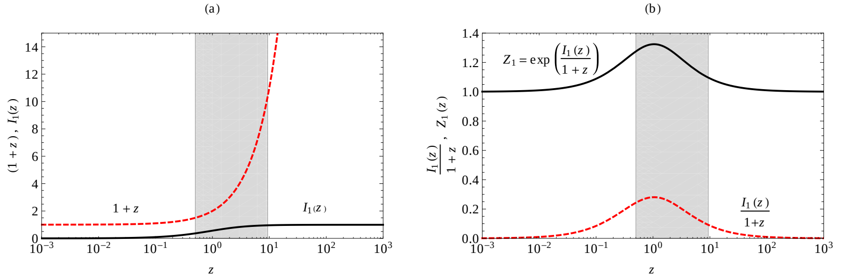
Rewrite it as

$$N_{z,1}(E_0, z) = N_0 [\mathcal{Z}_1(z)]^{-\gamma/E_0}$$

with $\gamma \equiv \kappa L_H$ and

$$\mathcal{Z}_1 = \exp\left(\frac{I_1(z)}{1+z}\right) \quad , \quad I_1(z) = \frac{L(z)}{L_H}$$

$\mathcal{Z}_1$ carries all of the redshift dependence.



Problems with the naive solution

1. $\mathcal{Z}_1 (z \gtrsim 10) = 1$ implies that

$$N_{z,1} (z \gtrsim 10) \simeq N_0 \ ,$$

regardless of E_0, while one expects that the more energetic neutrinos should live longer than the less energetic ones.

2. The redshift suppression $\mathcal{Z}_1$ was expected to be a monotonically growing function of z, so that $N_{z,1} \propto 1/\mathcal{Z}_1$ falls with z, since neutrinos from more distant sources should have more time to decay.

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The *proper* solution

Step 1: replace $E = (1+z)\,E_0$ in the differential equation:

$$\frac{dN}{dL} = -\lambda(z)\,N = -\frac{\kappa}{E_0\,(1+z)}N$$

Step 2: replace $dN/dL = (dN/dz)\,(dz/dL)$, so that

$$\frac{dN}{dz} = -\frac{\kappa}{E_0}\frac{dL}{dz}\frac{N}{1+z}$$

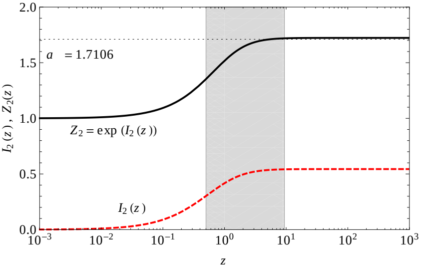
The proper solution is

$$N_{z,2}(E_0, z) = N_0\,[\mathcal{Z}_2(z)]^{-\gamma/E_0} ,$$

with

$$\mathcal{Z}_2(z) = e^{I_2(z)} \quad , \quad I_2(z) = \frac{1}{L_H}\int_0^z \frac{1}{(1+z')}\frac{dL}{dz}(z')$$

Behaviour of the proper solution



$$\mathcal{Z}_2(z) \simeq a + be^{-cz} \quad \text{with} \quad \begin{cases} a = 1.71 \\ b = 1 - a = -0.71 \\ c = 1.27 \end{cases} \quad \text{in } \Lambda\text{CDM}$$

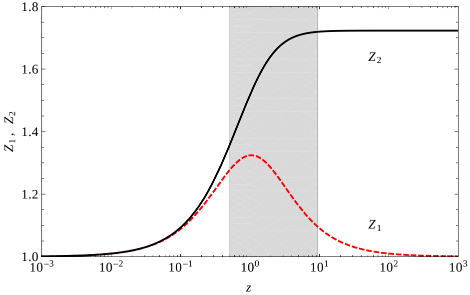
$$\Rightarrow N_{z,2}(z) \simeq \left(a + be^{-cz}\right)^{-\gamma/E_0}$$

Surprisingly, at $z \gg 1$,

$$N_{z,2}(z \gg 1) \simeq N_0 a^{-\gamma/E_0} \neq 0 \,,$$

i.e., the decay is *not* automatically complete at large z, because $L(z) \leq L_H$.

Different redshift dependence than in the naive approach:



$N_{z,2} \simeq \left(a + be^{-cz}\right)^{-\gamma/E_0}$ behaves as expected:

- It falls with z (down to $N_{z,2} \simeq N_0 a^{-\gamma/E_0}$), since for higher z there is more time for the decay to occur
- It grows with E_0, since the decay constant $\lambda \propto E_0^{-1}$

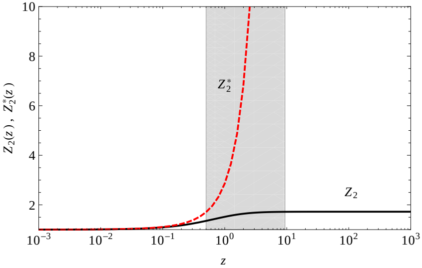
$N_{z,2}$ actually behaves like a (physically) proper solution!

Why is the decay not necessarily complete at high z?

- According to the usual, non-redshift dependent, solution,

$$N(E, L) = N_0 e^{-\kappa L/E} \ ,$$

  the population (for fixed E) disappears if $L \gg 1$, i.e., $N(E, L \gg 1) \to 0$.
- In contrast, $N_{z,2}(z \gg 1) \simeq N_0 a^{-\gamma/E_0}$
- Reason: light-travel distance $L(z)$ is bounded by L_H, which makes $\mathcal{Z}_2$ bounded (since $N_{z,2}(z) \propto \mathcal{Z}_2^{-1}(z)$)
- If we instead use $L(z) =$ luminosity distance (unbounded), we obtain an unbounded $\mathcal{Z}_2^*$ and $N^*_{z,2}(z \gg 1) \to 0$:



Proper condition for complete decay

From

$$N_{z,2}(E_0, z) = N_0 \left[\mathcal{Z}_2(z)\right]^{-\gamma/E_0}$$

since $\mathcal{Z}_2(z \gg 1) \nrightarrow \infty$, the condition for complete decay is $\gamma/E_0 \gg 1$, or,

$$\kappa^{-1} \equiv \frac{\tau_0}{m} \ll \frac{L_H}{E_0} \approx \frac{4 \times 10^8}{E_0\,[\text{GeV}]}\ \text{s eV}^{-1} .$$

In the relevant range, $E_0 \in \left[10^5, 10^{12}\right]$ GeV, this can be satisfied together with the current bounds on τ_i/m_i.

$\therefore$ complete decay is still allowed,
but it will not be due to very long baselines

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Flavour-transition probability

Average probability for astrophysical neutrinos:

$$P_{\alpha\beta}(E_0, z) = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 [\mathcal{Z}_2(z)]^{-\gamma/E_0} ,$$

with U the PMNS matrix.

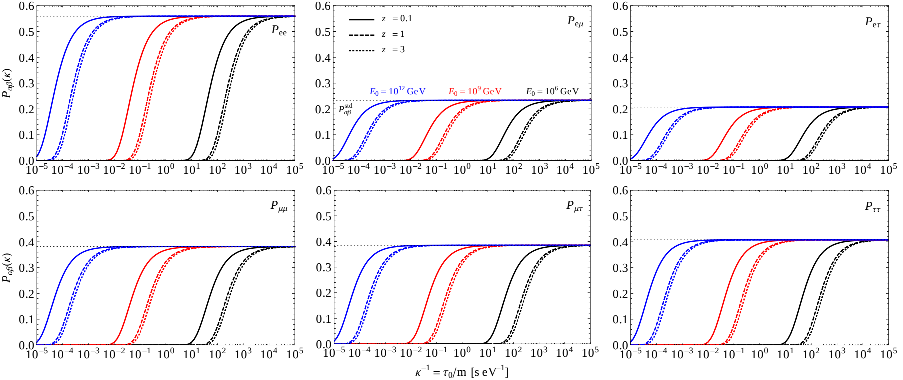
Note that

$$\frac{\gamma}{E_0} \simeq 4 \times 10^8 \frac{\kappa \left[\text{eV s}^{-1}\right]}{E_0 \,[\text{GeV}]} .$$

We have fixed

$$\sin^2\theta_{12} = 0.31 , \quad \sin^2\theta_{23} = 0.51 , \quad \sin^2\theta_{13} = 0.01 , \quad \delta_{\text{CP}} = 0$$

Implicit assumption: *all* mass eigenstates decay with $\kappa_i \equiv \kappa = m/\tau_0$



- at low κ^{-1}, the condition for complete decay is satisfied, so $P_{\alpha\beta} \to 0$
- at high κ^{-1}, $\gamma/E_0 \to 0$, so $P_{\alpha\beta} = P_{\alpha\beta}^{\rm std}$
- $\lambda(z) = \kappa/[E_0(1+z)]$, so higher E_0 require lower κ^{-1} for decay to occur
- deviations from $P_{\alpha\beta}^{\rm std}$ occur for $10^{-5} \lesssim \kappa^{-1}\left[\text{s eV}^{-1}\right] \lesssim 10^{5}$
- range is discarded for ν_1, so maybe ν_e's are not good probes of decay

Flavour composition at Earth

Flavour composition at source:

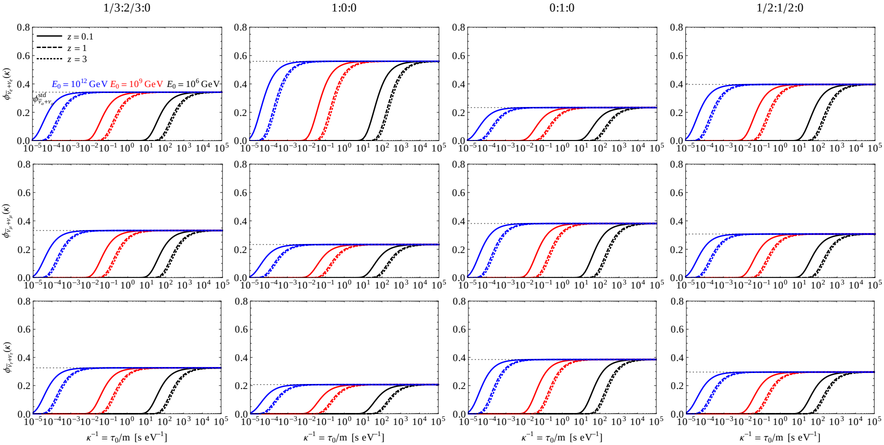
$$\phi^0_{\nu_e+\bar{\nu}_e} : \phi^0_{\nu_\mu+\bar{\nu}_\mu} : \phi^0_{\nu_\tau+\bar{\nu}_\tau}$$

At Earth, after mixing,

$$\phi_{\nu_\alpha+\bar{\nu}_\alpha}(E_0, z) = \sum_{\beta=e,\mu,\tau} P_{\beta\alpha}(E_0, z)\, \phi^0_{\nu_\beta+\bar{\nu}_\beta}$$

Four production scenarios:

1 : 2 : 0	pion beam source
1 : 0 : 0	neutron beam source
0 : 1 : 0	muon damped source
1 : 1 : 0	muon beam source


1/3:2/3:0
1:0:0
0:1:0
1/2:1/2:0
z = 0.1
z = 1
z = 3
E0 = 10^12 GeV E0 = 10^9 GeV E0 = 10^6 GeV
κ^−1 = τ0/m [s eV^−1]

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Point sources

- For now, we will consider only the classical fireball model
- Neutrino flux at Earth from a single source (all flavours):

$$E_0^2 F_\nu(E_0) = f_\nu \times \begin{cases} \left(\frac{E_0}{E_{\nu,\text{break}}}\right)^{-\alpha_\nu} & , \quad E_0 < E_{\nu,\text{break}} \\ \left(\frac{E_0}{E_{\nu,\text{break}}}\right)^{-\beta_\nu} & , \quad E_{\nu,\text{break}} \leq E_0 < E_{\nu,\mu} \\ \left(\frac{E_0}{E_{\nu,\text{break}}}\right)^{-\beta_\nu} \left(\frac{E_0}{E_{\nu,\mu}}\right)^{-2} & , \quad E_\nu \geq E_{\nu,\mu} \end{cases} ,$$

with f_ν, $E_{\nu,\text{break}}$, $E_{\nu,\mu}$, α_ν, and β_ν calculated from the observed GRB parameters z, Γ, $\varepsilon_{\gamma,\text{break}}$, t_v, L_γ^{iso}.

- Flux of $\nu_\mu + \bar{\nu}_\mu$ at Earth:

$$E_0^2 \phi_{\nu_\mu}(E_0) = E_0^2 F_\nu(E_0)\, \phi_{\nu_\mu+\bar{\nu}_\mu}(E_0, z) ,$$

where decay might affect the μ-flavour fraction, $\phi_{\nu_\mu+\bar{\nu}_\mu}(E_0, z)$

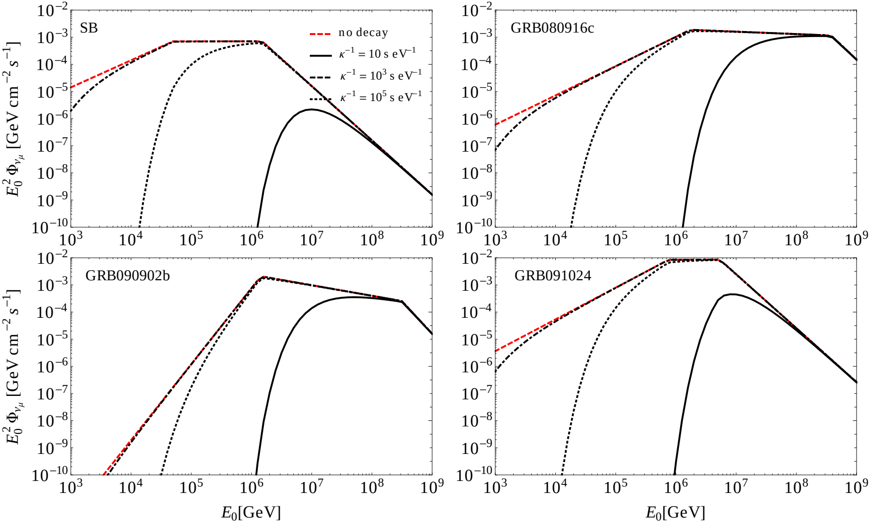
Four sample bursts:

	SB	**GRB080916c**	**GRB090902b**	**GRB091024**
α	1	0.91	0.61	1.01
β	2	2.08	3.80	2.17
$\epsilon_{\gamma,\mathrm{break}}$ [MeV]	1.556	0.167	0.613	0.081
Γ	$10^{2.5}$	1090	1000	195
t_v [s]	0.0045	0.1	0.053	0.032
T_{90} [s]	30	66	22	196
z	2	4.35	1.822	1.09
$\mathcal{F}_\gamma$ [erg cm^{-2}]	$1 \cdot 10^{-5}$	$1.6 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$	$5.1 \cdot 10^{-5}$
L_γ^{iso} [erg s^{-1}]	10^{52}	$4.9 \cdot 10^{53}$	$3.6 \cdot 10^{53}$	$1.7 \cdot 10^{51}$

SB: standard burst

S. Hümmer, P. Baerwald, and W. Winter, Private note on neutrino emission from GRBs in the fireball model

$$\kappa^{-1} = \tau_0/m$$

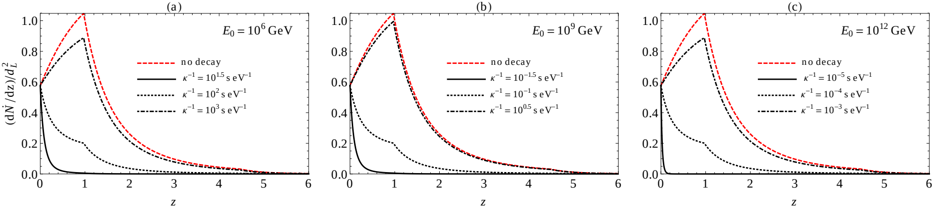


Diffuse flux

Assume that the redshift distribution of the GRB population N_s follows the (corrected) star formation rate, i.e.,

$$\frac{d\dot{N}_s}{dz} \propto (1+z)^{1.2} \, \dot{\rho}_* \, (z)$$

Relative contribution $L^{-2}(z)\, d\dot{N}_s/dz$ of GRBs at different redshifts (normalised to the value at $z = 1$):



Original (no decay) analysis in P. Baerwald, S. Hümmer, and W. Winter, Astropart. Phys. 35, 508 (2012) [1107.5583]

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To do

- Repeat the flux calculation using the revised fireball model with IceCube-40 from S. Hümmer, P. Baerwald, and W. Winter, Phys. Rev. Lett. 108, 231101 (2012) [1112.1076]:
  - compare fluxes in the naive and proper approaches
  - extrapolate to IceCube-86?
  - what is the impact of decay on the UHE neutrino flux bounds?
- Explore three scenarios of decay:
  - only ν_1 is stable (in agreement with SN1987A)
  - all ν_i are unstable (currently assumed)
  - all ν_i are stable (standard scenario without decay)

Backup slides

Light-travel distance

Hubble parameter in the ΛCDM cosmology:

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda} \,,$$

with $H_0 = 70.5$ km s^{-1} Mpc^{-1}, $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$.

The light-travel, or lookback, distance is an actual measure of a particle's pathlength:

$$L(z) = L_H \int_0^z \frac{dz'}{(1+z')\, h(z')} \leq L_H \,,$$

with $h(z) \equiv H(z)/H_0$ and the Hubble length $L_H \equiv c/H_0 \approx 3.89$ Gpc.

Also, from the definition,

$$\frac{dL}{dz} = \frac{L_H}{(1+z)\, h(z)} \,.$$

Approximate form of the naive solution

It is not as simple as for the proper solution:

$$\mathcal{Z}_1(z) \simeq p + qz + rz^2 + se^{-tz} \quad \text{with} \quad \begin{cases} p = 1.40 \\ q = -0.06 \\ r = 3.30 \times 10^{-3} \\ s = -0.41 \\ t = 3.09 \end{cases} \quad \text{in } \Lambda\text{CDM}$$

This describes the peaked behaviour of $\mathcal{Z}_1$.

So,

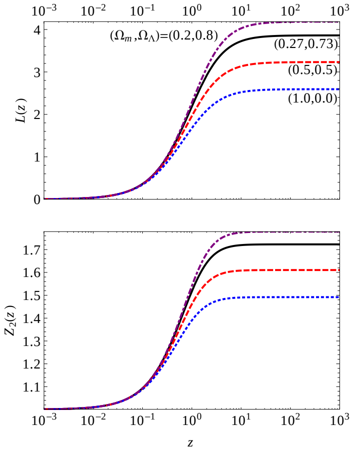
$$N_{z,1}(E_0, z) \simeq \left(p + qz + rz^2 + se^{-tz}\right)^{-\gamma/E_0} ,$$

but the behaviour with z is not transparent, unlike the approximate expression for $N_{z,2}$.

Effect on $N_{z,2}$ of the choice of cosmology

$$N_{z,2}(E_0, z) \simeq N_0 \left(a + be^{-cz}\right)^{-\gamma/E_0} \xrightarrow{z \gg 1} N_0 a^{-\gamma/E_0}$$

The values of a and c depend on the choice of cosmology, e.g.,



- $L^{(0.2,0.8)} > L^{\Lambda\text{CDM}} > L^{(0.5,0.5)} > L^{\text{EdS}}$ so, ν's have more time to decay in a $(0.2, 0.8)$ cosmology than in an EdS cosmology
- this translates into $\mathcal{Z}_2^{(0.2,0.8)} > \mathcal{Z}_2^{\Lambda\text{CDM}} > \mathcal{Z}_2^{(0.5,0.5)} > \mathcal{Z}_2^{\text{EdS}}$
- in other words, the effective Hubble lengths $L_H^{(0.2,0.8)} > L_H^{\Lambda\text{CDM}} > L_H^{(0.5,0.5)} > L_H^{\text{EdS}} \ldots$
- ... and the values of a for the cosmologies follow the same ordering

Decay using luminosity distance

The luminosity distance is defined as

$$L(z) = (1+z)\, L_H \int_0^z \frac{dz'}{h(z')} ,$$

and so

$$\frac{dL}{dz} = L_H \left(\int_0^z \frac{dz'}{h(z')} + \frac{1+z}{h(z)} \right) .$$

Replacing in

$$\frac{dN}{dz} = -\frac{\kappa}{E_0} \frac{dL}{dz} \frac{N}{1+z}$$

and solving yields

$$N^*_{z,2}(E_0, z) = N_0 \left[\mathcal{Z}^*_2(z)\right]^{-\beta/E_0} ,$$

where

$$I^*_2(z) \equiv \int_0^z \frac{dz'}{1+z'} \int_0^{z'} \frac{dz''}{h(z'')} + \int_0^z \frac{dz'}{h(z')} ,$$

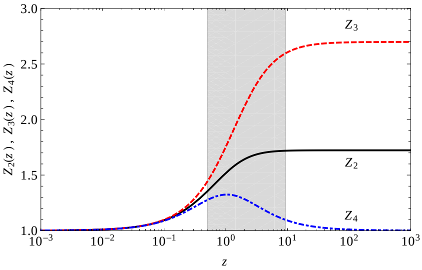
$$\mathcal{Z}^*_2(z) \equiv e^{I^*_2(z)} .$$

Hybrid solutions

To gain insight into the redshift dependence of the decay, we have explored two *hybrid* approaches:

- $N_{z,3}$: consider $L = L(z)$ before solving and do not consider the z-dependence of the energy (i.e., $E = E_0$)
- $N_{z,4}$: replace $E = (1+z)E_0$ in $N_{z,3}$

Common functional form: $N_{z,n}(E_0, z) = N_0 [\mathcal{Z}_n(z)]^{-\gamma/E_0}$



- $\mathcal{Z}_4$ is pathological (like the naive $\mathcal{Z}_1$)
- $\mathcal{Z}_3$ has the same shape as the proper $\mathcal{Z}_2$, but higher asymptotic value
- reason: in $\mathcal{Z}_2$, the z-dependence of E_0 raises it (by $(1+z)$), so that $\lambda(z)$ is lower for neutrinos that come from farther away and hence a larger population survives
- such a reduction of λ is not present in $\mathcal{Z}_3$, so it is higher and the surviving population $N_{z,3}(z) \propto \mathcal{Z}_3^{-1}(z)$ is smaller